Information Aggregation using Prediction Markets and Peer Prediction, A Survey

Moin Hussain Moti
Machine Learning Lab
International Institute of Information Technology,
Hyderabad, India
moinhussain.moti@students.iiit.ac.in

Sujit Gujar
Machine Learning Lab
International Institute of Information Technology,
Hyderabad, India
sujit.gujar@iiit.ac.in

***Abstract*—This survey focuses on how Prediction Market based Mechanisms have evolved as an efficient way to elicit information from agents with minimal assumptions on agents' distribution of prior beliefs on outcomes/products. Agents, in general, may have ulterior incentives to lie in order to gain higher payoffs, therefore it becomes imperative to apply game theory and induce games where the best strategy for agents is to provide truthful information.**

CONTENTS

I. INTRODUCTION

The world at present has evolved tremendously since the dawn of the 20^{th} century. The main reason that propelled this growth was the Internet. The Internet connected people across all domains and became an interactive platform for all worldly needs. In a decade or so, the whole world went online. Although it brought with it lot of benefits, one big disadvantage was the establishment of trust. Trust comes with experience and given the vastness of the Internet, it is unimaginable for any person to hold experience over more than a few domains. One possible solution to this problem is aggregating people's experience into some of the ratings which can be used by everyone on the network. The biggest problem with aggregating information is that how do you make sure the participants are telling the truth. For example, a particular person may have some grudge against a particular agency and therefore will want to report bad experience irrespective of the service of the agency. Therefore, truthful elicitation of information is very important. But people are rationally very cautious over the information they share on the Internet since it is public information, hence it is important to elicit truthful responses from agents using minimal information from them. There are two main types of mechanisms prevalent in the literature for these problems, namely, Peer Prediction and Bayesian Truth Serum. We focus only on Prediction Markets in this report.

We first give a classification of papers based on their contribution in an evolutionary manner, also presenting the similarities and differences simultaneously and then we present a summary of each paper in the subsequent sections.

II. CLASSIFICATION OF PAPERS

A. Fundamental Level

Papers:

- Eliciting Informative Feedback: The Peer-Prediction Method
- An Incentive Compatible Reputation Mechanism

Key Points:

1) These papers require prior distribution of agents' private beliefs.
2) The mechanisms are incentive compatibles.

3) Truthful reporting is Nash Equilibrium but there also exists other Nash Equilibriums which imply lying. Therefore, no strict Nash Equilibrium for truthful reporting.
4) No extensibility to non-binary signals is proposed in the paper.
5) The first paper used strictly proper scoring rules for payoff schemes which are also used in other papers studied in this report.

B. First Upgrade over Fundamental Level

Papers:

- Enforcing Truthful Strategies in Incentive Compatible Reputation Mechanisms

Key Points:

1) Made modifications to the above 2 papers such that truthful reporting becomes a strict Nash Equilibrium.
2) Increased cost with respect to above 2 papers as the mechanism compares the reports with expected trusted reports and trusted reports are expensive.
3) No extensibility to non-binary signals is proposed in the paper.
4) The mechanisms suggests no modification to the payoff schemes but only to the reports compared against in the payoff schemes, the original mechanisms can continue with their respective payoff schemes.

C. Less Assumptive

Papers:

- Robust Incentive-Compatible Feedback Payments
- Peer Prediction with private beliefs
- Peer Prediction without a Common Prior

Key Points:

1) The first paper in the list is robust to some degree of private information while the next two papers do not require agents' prior distribution of private beliefs.
2) The last paper is a significant upgrade because it does not assume the availability of any common prior distribution of beliefs.
3) However, these mechanisms do have restrictions on the information structure of the agents. These mechanisms require all the agents to have the same signal space and their signals are identically and independently drawn conditional on the realized event outcome.
4) No extensibility to non-binary signals is proposed in the paper.
5) The first paper uses Linear Programming to get the optimal payment distribution scheme while the next two papers use strictly proper scoring rules (Quadratic Rule).

D. Least Assumptive

Papers:

- Elicitability and Knowledge-Free Elicitation with Peer Prediction

Key Notes:

1) This model does not place any restriction on the information structure as long as it is a valid probability distribution over the signal space and outcome space.
2) It can strictly truthfully elicit private information for both small and large population settings for both binary and non-binary private signals.
3) This mechanism uses Strictly Proper Scoring Rule (Quadratic Rule)

III. Research Paper Index

Refer this report for a brief introduction of the research paper surveyed.

Table Index			
Paper	**Appeared in**	**Problem Addressed**	**Novel Approach**
Fundamental Level			
Eliciting Informative Feedback: The Peer-Prediction Method [1]	Journal, Management Science, 15	Elicit truthful information from agents using peer-prediction mechanism	The mechanism induces a Nash Equilibrium where the best strategy for the agent is to report truthfully. It does so by using Strictly Proper Scoring Rule
An Incentive Compatible Reputation Mechanism [2]	IEEE 03	Develop an incentive compatible mechanism to elicit truthful feedback from agents about other agents and based on the feedback, update their reputation	Model a game which induces a Nash Equilibrium where the best strategy is to report truthfully
First Upgrade over Fundamental Level			
Enforcing Truthful Strategies in Incentive Compatible Reputation Mechanisms [3]	WINE 05	Mechanisms proposed so far have more than one Nash Equilibria which implies lying	Rate reports against expected trusted reports to induce Strict Nash Equilibrium
Less Assumptive			
Robust Incentive-Compatible Feedback Payments [4]	Automated Negotiation and Strategy Design for Electronic Markets 2007	Make information elicitation robust with respect to private beliefs of agents to some degree	Keep some slack variable in the prior beliefs of agents to account for external private beliefs of agents
Peer Prediction with private beliefs [5]	EC 12	Design a peer prediction mechanism that works even when agents have private beliefs about the product unknown to the mechanism	Take agent's prior and posterior beliefs and determine agent signal based on that and then use the inferred signal to generate payoffs
Peer Prediction without a Common Prior [6]	EC13	Design a peer prediction based information elicitation mechanism which does not assume agents to have prior common belief	Take agent's prior belief and instead of posterior belief, take agent's observed signal. Rate the report accordingly using Strictly Proper Scoring Rule to induce Nash Equilibrium where truthful reporting is the best strategy
Least Assumptive			
Elicitability and Knowledge-Free Elicitation with Peer Prediction [7]	AAMAS 14	Allow agents to have private beliefs and relax same signal and outcome space for all agents	Ask the agent to make 2 submission, one for information report and one for prediction report where information report involves agent submitting his observed signals and prediction report involves agent submitting his prediction i.e. joint distribution over the signals of all agents except himself and the one other agent specified later.

IV. FUNDAMENTAL LEVEL

A. Eliciting Informative Feedback: The Peer-Prediction Method

Model

Given a product whose type t (product's quality which remains constant throughout the experiment and can be expressed in the form of a tuple if required) is drawn from a set of possible types T, a number of agents interact with the product (in general sequentially) and form their respective private perception of the product expressed as a signal s from a set of signals S (S is a finite set of cardinality M). After every interaction, the agents are required to announce the feedback to the system about the signal she has observed.

Mechanism

The Mechanism assumes that rater's utilities are linear in points and that the prior belief of the type of the product is public information. Let $p(t)$ represent the prior probability that the product is of type t. Since the total possible types of the product is finite, we can write $\sum_{t \in T} p(t) = 1$. The signals are conditioned on the hidden type t of the product and are independent and identically distributed. It can be denoted by $f(s_i|t)$, where $f(s_i|t) > 0$ and $\sum_{s_i \in S} f(s_i|t) = 1$. In the sequential setting, as and when the system receives a report $r \in S$, the posterior belief of the product type is updated as:

$$p(t|r) = \frac{f(r|t).p(t)}{Pr[r]} \tag{1}$$

where $Pr[r] = \sum_{t \in T} f(r|t).p(t)$ is the probability of observing the signal $r \in S$.

Payment: It can be interpreted as every agent's report is used to update the probability distribution for the report of a future agent (we use the term *player* for the agent who is being rated and the term *rater* for the agent on whose report the player is being rated). The payment/score is based on the accuracy of the prediction of player's rating by the system (the posterior belief of the product's type) i.e. the comparison between the likelihood assigned to the rater's possible ratings and the rater's actual ratings.
Three strictly proper scoring rules have been mentioned in the paper, namely

- Quadratic Scoring Rule
- Spherical Scoring Rule
- Logarithmic Scoring Rule

Transfers based on a strictly proper scoring rule can be used to induce truthful revelation by a player as long as her private information is *stochastically relevant* for some other publicly available signal.
Stochastic Relevance: Random variable S^i is stochastically relevant for random variable S^j if and only if the distribution of S^j conditional on S^i is different for different realizations of S^i.

	C	D
C	R,R	S,T
D	T,S	P,P

TABLE I
N RATIONAL AGENTS INTERACT PAIRWISE IN AN ITERATED PRISONER'S DILEMMA GAME. THE FOLLOWING CONSTRAINTS ARE ASSIGNED TO THE PAYOFFS: T ¿ R ¿ P ¿ S

B. An Incentive Compatible Reputation Mechanism

Model

The last paper [1] is more relevant in terms of rating products while this paper is more about the reputation of agents in an interactive environment. Let there be two agents, agent A and agent B. Each agent has a binary action space i.e. 0,1. Each agent reports the action of the other agent. An agent can buy reputation information of the opposite agent from special agents known as *R-agents*. Reputation information about an agent is represented by last M reports, where M is some integer. Agent A can decide whether to enter into a game with agent B based on the reputation info bought from the R-agent. Each agent can submit his report about the opposite agent only to the R-agent from which they bought the opposite agent's reputation information.

Payment:: R-Agents pay the reputation report of agent A about agent B only if it matches the next report submitted about B. Apart from that each agent receives a separate payoff from the award matrix of the game with another agent as well.

V. FIRST UPGRADE OVER FUNDAMENTAL LEVEL

A. Enforcing Truthful Strategies in Incentive Compatible Reputation Mechanisms

This paper argues that while many schemes have Nash Equilibrium for truthful reporting, they also other Nash Equilibriums which imply lying. The paper proposes an easy solution to the problem that if the reports are rated against trusted reports, only one Nash Equilibrium which exists and that will be truthfully reporting.
The main challenge in this is that trusted reports are expensive to get. The solution to this is that the reports be scored against a trusted report with probability q and scored against an untrusted report with probability $(1 - q)$.

VI. LESS ASSUMPTIVE

A. Robust Incentive-Compatible Feedback Payments

Model

The model is same as that described in above sections.

Mechanism

The main difference is that instead of using strictly proper scoring rule functions, this paper proposes solving linear and conic optimization problems to get the scoring rule mechanism that results in optimal incentive compatible feedback payments.

$$\sum_{k=1}^{M} Pr[s_k|s_j](\tau(s_j,s_k) - \tau(s_h,s_k)) > \Delta(s_j,s_h); \forall s_j \neq s_h \in S$$
$$\sum_{k=1}^{M} Pr[s_k,s_j]\tau(s_j,s_k) > C; \forall s_j \neq s_h \in S$$

The optimal payment scheme $\tau(.,.)$ satisfying the above conditions is incentive-compatible. The optimal payment scheme minimizes the budget required by the reputation mechanism, and therefore solves the following linear program (i.e., linear optimization problem): The optimal payment scheme minimizes the budget required by the reputation mechanism and therefore solves the following linear program (i.e., linear optimization problem):

$$min \;\; W = \sum_{j=1}^{M} Pr[s_j](\sum_{k=1}^{M} Pr[s_k|s_j]\tau(s_j,s_k))$$

$s.t.$

$$\sum_{k=1}^{M} Pr[s_k|s_j](\tau(s_j,s_k) - \tau(s_h,s_k)) > \Delta(s_j,s_h); \forall s_j \neq s_h \in S$$
$$\sum_{k=1}^{M} Pr[s_k,s_j]\tau(s_j,s_k) > C; \forall s_j \neq s_h \in S$$
$$\tau(s_j,s_k) \geq 0; \forall s_j, s_k \in S$$

The paper makes the model robust by modifying the prior belief of the reputation mechanism to accommodate dynamic unknown private beliefs.
Let $(Pr[\theta])_{\theta\in\Theta}$ characterize the prior belief of the reputation mechanism and let $(Pr*[\theta] = Pr[\theta] + \eta_\theta)_{\theta\in\Theta}$ be the range of private beliefs the agents might have, where $\sum_{\theta\in\Theta}\eta_\theta = 0$ and $max(-\epsilon, -Pr[\theta]) \leq \eta_\theta \leq min(\epsilon, 1 - Pr[\theta]), \epsilon > 0$.
Using this modification in the all the equations, we can form a robust incentive-compatible feedback mechanism.

B. Peer Prediction with Private Beliefs

Previous papers [1], [2], [4] assumed that there exist a common prior held beliefs. This paper present prediction mechanisms which don't assume any common knowledge assumptions. Agents hold their private beliefs for the product.

Model

The signal set is binary, $S = l, h$ (analogous to low and high quality). Agent i provides the system with a prior belief and posterior belief. Based on the beliefs, the system infers agent's signals in the following way:

$$r^i = \begin{cases} h, & if \quad r_2^i > r_1^i \\ l, & if \quad r_2^i < r_1^i \end{cases} \tag{2}$$

where r_1^i and r_2^i are the prior and posterior belief respectively. Agent i's report is then rated against some other agent's report.

Payment:: A strictly proper rule (Quadratic Scoring Rule in this case) is used which uniquely maximizes the agent's expected payment through an honest report. Let agent i's prior and posterior belief be r_1^i and r_2^i respectively and agent j's implicit belief inferred by the system be r^j, then payment to agent i is calculated as:

$$R(r_1^i, r^j) + R(r_2^i, r^j) \tag{3}$$

where,

$$R(p,q) = \begin{cases} 2p - p^2, & if \quad q = h \\ 1 - p^2, & if \quad q = l \end{cases} \tag{4}$$

C. Peer Prediction without a Common Prior

This is a follow-up paper from the previous paper. The Model remains same but new mechanisms are introduced where agent i is rated against actual signals reports of agent j instead of signal belief reports of agent j.

Model

The model is similar to the one mentioned in the last section. The only difference is that instead of posterior signal belief report, the agent submits her signal report.

Mechanisms

Three different mechanisms are introduced and efficient increases from first to last.

1) Candidate SPP:

- Agent i submits her prior signal belief report $y_i \in [0,1]$ that another agent will receive a high signal.
- Agent i observes signal $S_i = s_i$.
- Agent i submits her signal report $x_i \in 0, 1$

The system now calculates the shadow posterior belief

$$y_i' = y_i'(y, x_i) = \begin{cases} y_i + \delta, & if \quad x_i = 1 \\ y_i - \delta, & if \quad x_i = 0 \end{cases} \tag{5}$$

where $\delta > 0$ is a parameter of the mechanism.

Agent i's score is then calculated as

$$u_i = R(y_i, x_j) + R(y_i', x_j) \tag{6}$$

where R is the quadratic scoring rule (mentioned in previous section) and x_j is the signal report of agent j.

2) SPP:

- Agent i submits her prior signal belief report $y_i \in [0,1]$ that another agent will receive a high signal.
- Agent i observes signal $S_i = s_i$.
- Agent i submits her signal report $x_i \in 0, 1$, and

The system now calculates the shadow posterior belief

$$y_i' = y_i'(y, x_i) = \begin{cases} y_i + \frac{\delta(1-2y_i)}{2} + \delta, & if \quad x_i = 1 \\ y_i + \frac{\delta(1-2y_i)}{2} - \delta, & if \quad x_i = 0 \end{cases} \tag{7}$$

where $\delta > 0$ is a parameter of the mechanism.

Agent i's score is then calculated as

$$u_i = R(y_i + \frac{\delta(1-2y_i)}{2}, x_j) + R(y_i', x_j) \tag{8}$$

where R is the quadratic scoring rule (mentioned in previous section) and x_j is the signal report of agent j.

3) Compact SPP:

- Agent i submits her prior signal belief report $y_i \in [0, 1]$ that another agent will receive a high signal.
- Agent i observes signal $S_i = s_i$.
- Agent i submits her signal report $x_i \in 0, 1$, and

Agent i's score is calculated as

$$u_i = R_q((1 - 2\delta)y_i + 2\delta x_i, x_j) \tag{9}$$

where $\delta > 0$ is a parameter of the mechanism, R is the quadratic scoring rule (mentioned in previous section) and x_j is the signal report of agent j.

VII. Least Assumptive

A. Elicitability and Knowledge-Free Elicitation with Peer Prediction

The paper proposes a mechanism that can elicit private information without any knowledge of the information structure of the agent. However, the mechanism does assumes the existence of a common prior in this paper. The mechanism allows individual agents to have different signal spaces.

Model

Let Ω denote the set of possible outcomes and true outcome is denoted by w. There are $n \geq 2$ rational and risk-neutral agents that are only interested in maximizing their payout. Each agent i observes a private signal θ_i drawn from a finite stet of all possible signals Θ_i. Agents are assumed to have common knowledge about the event and how it affects the prior distribution ($Pr[w, \theta]$) of their signals over the outcome space and signals spaces. $Pr[w, \theta]$ is referred as *Information Structure* ($\mathcal{I} \equiv \Delta(\Omega \times \Theta)$) in this paper. The mechanism places no restriction on the information structure as long as it is a valid probability distribution over Ω and Θ.

Mechanism

1) For every agent i, select 2 reference agents $h = (i-1) \mod n$, $j = (i+1) \mod n$. Now, every agent simultaneously reports a signal $x_i \in \Theta_i$ which can be different from his true signal θ_i, to the mechanism.
2) Agent i receives the information report of player h, x_h, from the mechanism, and then reports a joint distribution p^i over the signals of all agents except agents i and h; we call this the prediction report.

Payment:: Agent i's payoff is given by:

$$R_p(p^j, (x_{-(i,j)})) + R_p(p^i, (x_{-(h,i)})) \tag{10}$$

where R_p is strictly proper is any strictly proper scoring rule. The first term is called the information report and the second term is called the prediction report.

References

[1] Miller, Nolan, Paul Resnick, and Richard Zeckhauser. ”Eliciting informative feedback: The peer-prediction method.” Management Science 51.9 (2005): 1359-1373.

[2] Jurca, Radu, and Boi Faltings. ”An incentive compatible reputation mechanism.” E-Commerce, 2003. CEC 2003. IEEE International Conference on. IEEE, 2003.

[3] Jurca, Radu, and Boi Faltings. ”Enforcing truthful strategies in incentive compatible reputation mechanisms.” International Workshop on Internet and Network Economics. Springer, Berlin, Heidelberg, 2005.

[4] Jurca, Radu, and Boi Faltings. ”Robust incentive-compatible feedback payments.” Agent-Mediated Electronic Commerce. Automated Negotiation and Strategy Design for Electronic Markets. Springer, Berlin, Heidelberg, 2007. 204-218.

[5] Witkowski, Jens, and David C. Parkes. ”Peer prediction with private beliefs.” Proceedings of the 1st Workshop on Social Computing and User Generated Content (SC 2011). 2011.

[6] Witkowski, Jens, and David C. Parkes. ”Peer prediction without a common prior.” Proceedings of the 13th ACM Conference on Electronic Commerce. ACM, 2012.

[7] Zhang, Peter, and Yiling Chen. ”Elicitability and knowledge-free elicitation with peer prediction.” Proceedings of the 2014 international conference on Autonomous agents and multi-agent systems. International Foundation for Autonomous Agents and Multiagent Systems, 2014.

[8] Faltings, Boi, and Goran Radanovic. ”Game Theory for Data Science: Eliciting Truthful Information.” Synthesis Lectures on Artificial Intelligence and Machine Learning 11.2 (2017): 1-151.