



## Are you game for Game Theory?



### Why Are Our Cities Dirty Even though Everyone Likes It Clean?



#### Tragedy of Commons

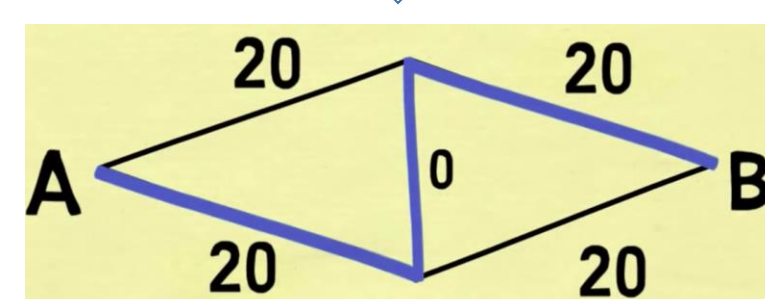
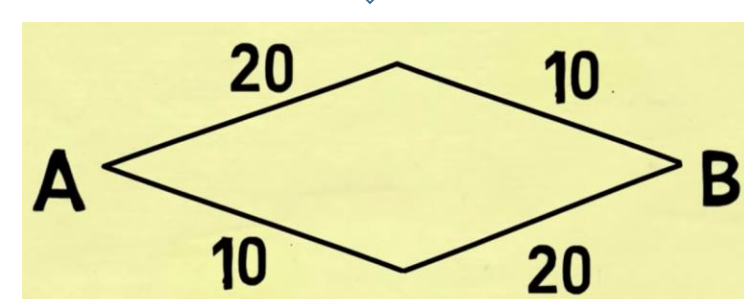
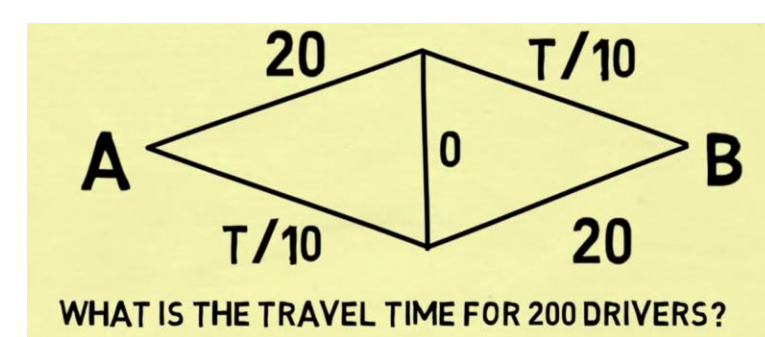
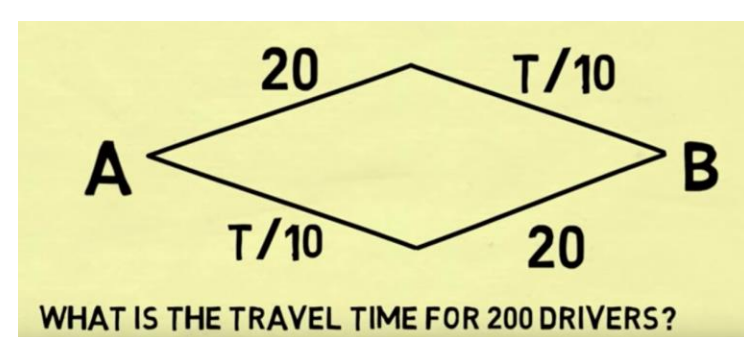
The marginal utility gained by an individual against his efforts to achieve common good is low. Such rationality leads to socially disadvantageous outcome

- Consider 2 players, A and B, trying to keep a surrounding clean
- A utility of 50 is achieved by BOTH players for every player that keeps the surrounding clean
- A player incurs a utility of -60 in his efforts to keep surrounding clean

	B keeps it clean	B keeps it dirty
A keeps it clean	40,40	-10,50
A keeps it dirty	50,-10	0,0

- When both A and B acts in self-interest, it leads to a dirty city while it would have been better off for both if they kept it clean

### Do More Roads Always Mean Lesser Traffic Congestion? NO



Adding a high link road as shown increases the overall commute time for the 200 drivers travelling from A-B, as their individually rational choice leads to a socially disadvantageous outcome. This is called the **Braess' Paradox**

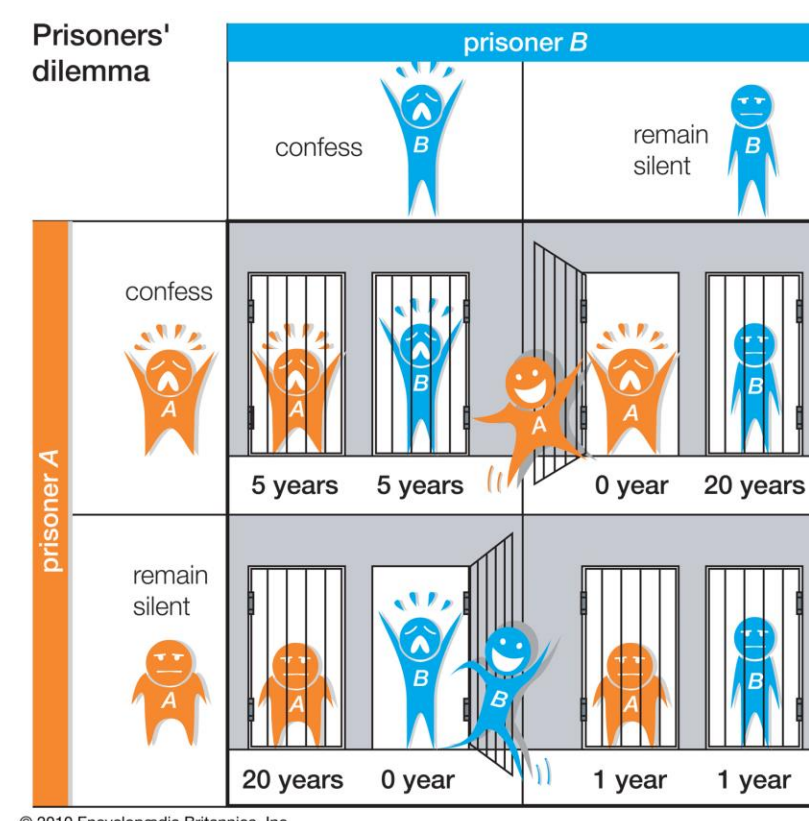
### GAME THEORY

Models the conflicts and cooperation between rational and intelligent agents

#### Rational and Intelligent



#### Utilities and Strategies



- Agents A and B
- Utilities = - time spent in prison (-5, -5), (0, -20), (-20, 0), (-1, -1)
- Strategies = {(C, C), (C, RS), (RS, C), (RS, RS)}

What should A and B do?

1. B confesses

A will confess

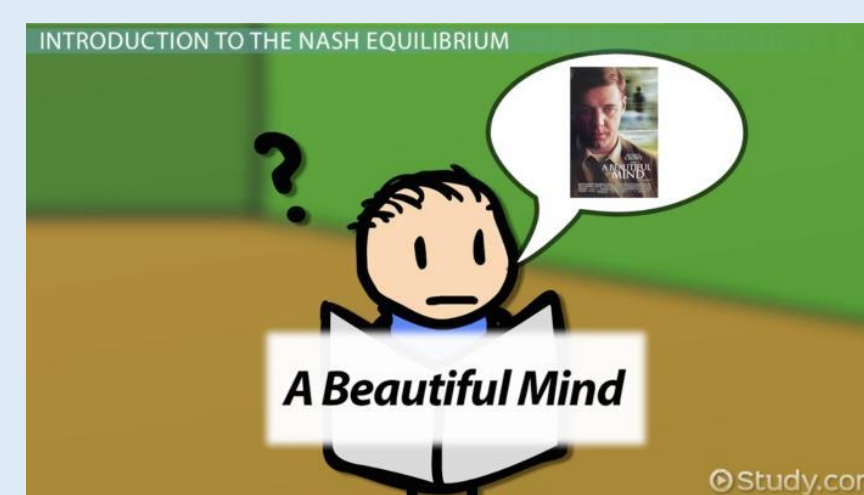
2. B remains silent

A will confess

Both 1 and 2 have same answer Confess is a Dominant Strategy

Similarly for B. Hence (C,C) is Dominant Strategy Equilibrium

1	2	
	NC	C
NC	-2, -2	-10, -1
C	-1, -10	-5, -5



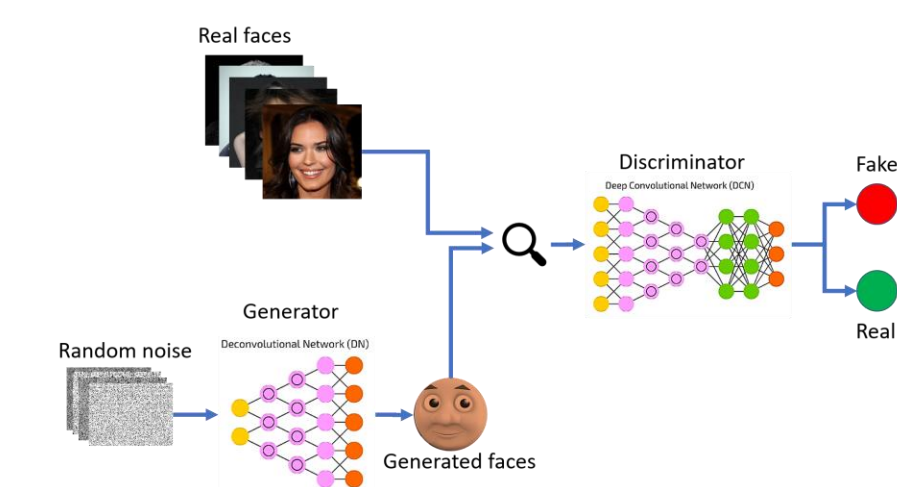
No Dominant Strategies !!  
So which strategies would the players choose?

**NASH EQUILIBRIUM**  
No incentive to deviate from (C, C)

Will There Always be Nash equilibrium?  
(1,-1) (-1, 1)  
(-1, 1) (1, -1)

### Game Theory to the rescue in complex scenarios !!

#### Two player zero sum games with infinite strategy space



- Players - Generator (G) and Discriminator (D)
- Strategies - Weights of D and G (Can we write it as a matrix ??)

EQUILIBRIUM !!

Or

What are the optimal weights?

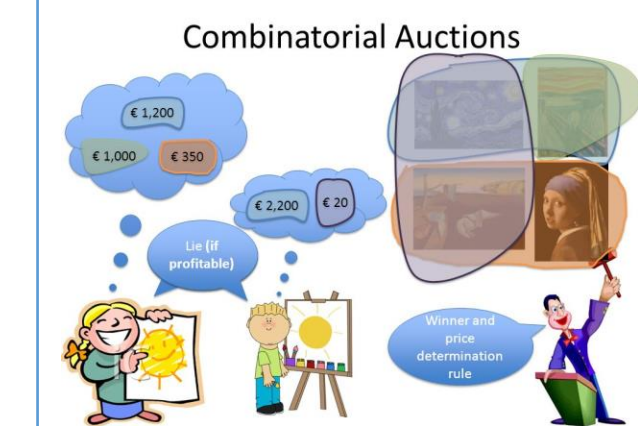
How do we find those weights in finite time? (PPAD Complete)

#### Incomplete Information (Bayesian Games)

Firms A and B in market. A wants to renovate. Private knowledge {High, Low investment}. B wants to enter the market

	High-investment cost		Low-investment cost	
	Enter [y]	Refrain [1-y]	Enter [x]	Refrain [1-x]
Modernize	0,-2	4,0	3,-2	7,0
Status quo	4,2	6,0	4,2	6,0

#### Sealed-bid auctions



The value for goods is private. What should be the allocation and payments?

### Differential Games

(Optimal Control + Game Theory)

#### CAR 1 (red) vs CAR 2 (blue)

Control Variables  
Accelerator pedal positions

Fuel Consumption Rates:  $y_1(t), y_2(t)$   
Fuel Cost:  $p_1, p_2$   
 $c = (c_1(t), c_2(t))$

Price Money: M, Total Time: T

Positions:  $r_1(t), s_1(t)$   
 $r_2 = r_1'(t), s_2 = s_1'(t)$

State Variables

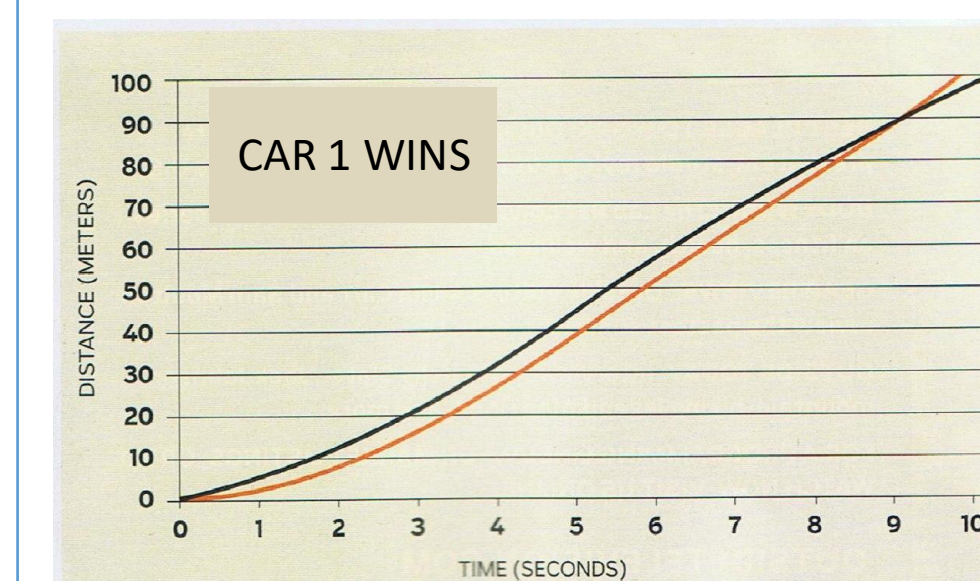
$z = (r_1, r_2, s_1, s_2)$   
 $z' = f(t, z, c)$

Payoff for player 1

$$M + \int_0^T (-y_1 * p_1) dt$$



Applications  
Economics, Military, etc



Distance Vs Time, straight line for first player